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WIND SHEAR MODELING AND DETECTION: AN ANALYSIS IN TERMS OF A FIRST-PASSAGE PROBLEM

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ABSTRACT

An approach is proposed for extracting information from airborne Doppler radar returns that can be used to determine the presence of wind shear in commercial aviation. Issues that are discussed include modeling in terms of stochastic differential equations, estimation in terms of minimummean-square-error filters, and detection in terms of a first passage problem. Some discussion of the hardware architecture needed to process the data in a timely and efficient manner is also included.

1. INTRODUCTION

The problem of wind shear in commercial aviation has always existed, but only recently has it been given the attention it deserves. In the last fifteen years, it has been listed as the cause of several accidents involving large carriers, with the most recent being the crash of Delta Airlines Flight 191 in Dallas on August 2, 1985 that claimed the lives of 137 people.

The phenomenon can be loosely described as a sudden downburst of wind at a velocity of aproximately 40 meters/second which, upon impact with the ground, results in a horizontal component of wind velocity with high magnitude, and with direction that abruptly changes as one moves through it at an altitude close to the ground. Thus it poses a severe hazard to aircraft who encounter this so-called microburst (see Fujita [1]) in either the take-off or landing phase of flight, in that the aircraft first experiences a high head wind causing lift followed suddenly by a high tail wind causing descent. Therefore, with little altitude in which to maneuver, pilots are unable to recover control and disaster results.

The many causes of wind shear are discussed in report [2]; also see Doviak and Zrnic [3,chap. 9]. These range from wet microbursts that occur in the wake of thunderstorms to dry microbursts that occur in more tranquil climatic conditions. Research in this area continues to gain a better understanding of the physics involved.

The above report also discusses various methods for detecting the presence of wind shear and alerting the pilot in a timely fashion. Among these is the use of ground based radars such as the system called NEXRAD, a planned network of Doppler radars to be operated by the National Weather Service, and a proposed Doppler radar system called TDWR that would be placed specifically at principal airports. The report also proposes the use of airborne Doppler radars, and it is this topic that we wish to explore in this paper.

To be more precise, we shall propose a method for extracting information from Doppler radar returns that can be used to determine the presence of wind shear. Issues to be discussed will include modeling in terms of stochastic differential equations, estimation in terms of minimum-mean-square-error (MMSE) filters, and detection in terms of a first-passage problem. In addition, we shall include some discussion of the hardware architecture needed to process the data in a timely and efficient manner.

2. MATHEMATICAL DESCRIPTION

The method of solution we shall investigate is based on the classical disruption problem. In the mathematics literature, this is sometimes called the first-passage problem or the exit problem. More precisely, one monitors the evolution of a stochastic process and attempts to determine the first time the process reaches a boundary or exceeds a threshold. In terms of detecting wind shear, the first time the threshold is exceeded will correspond to the onset of a microburst. Our formulation will be based on a paper by Davis [4] who shows how nonlinear filtering techniques can be useful in the detection process. We now proceed to describe the radar signal returns, the state space model, and the filtering and detection algorithm.

Radar Signal Returns

The radar signals used in weather applications are composites of signals from a very large number of scatterers (e.g., hydrometeors) each of which can be considered a point target; collectively they describe a so-called distributed target. After the round trip propagation delay, echoes due to one pulse are continuously received over a time interval equal to twice the time it takes the pulse to propagate across the volume containing

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the scatterers. The echoes can be expressed as (see, e.g., Van Trees [5])

$$r(\tau_s) = r(2r_v/c)$$
$$= \frac{1}{\sqrt{2}}[\sum_i A_i W_i e^{-4\pi r_i/\lambda}]e^{-j4\pi r_v/\lambda}$$

where $\tau_s = 2r_v/c$ is the so-called range time, r_v is the range resolution volume, r_i is the incremental range within the volume, $|A_i|/\sqrt{2}$ is the echo amplitude of the i^{th} scatterer located at range $r_v + r_i$, and W_i is the corresponding range weighting function. For a short pulse duration, however, this equation does not contain any Doppler shift information because weather targets move at relatively slow speed (r_i is constant during the time scatterer i is illuminated by the pulse). Therefore, in order to include Doppler shift information needed for wind shear analysis, one would have to look at returns from a number of pulses where each return is of the above form and consists of reflections at a number of range gates. By proper sampling, then, a sequence of complex video samples $r(kT_s)$ separated by the pulse repetition time T_s can be obtained for each and every range gate. This sequence would consist of a signal in additive noise, i.e.,

$$r(kT_s) = S_k e^{j\omega_d kT_s} + V_k, \quad k = 0,1,\ldots,M-1$$

where w_d is the Doppler shift and M is the number of returns considered at one time. The fundamental difference between these equations is that the former describes the reflection process in space (range time) while the latter describes it in time (sample time) at one specific range. Because our analysis will be based on the sample time sequences that contain the Doppler shift information, the use of dynamical models that incur abrupt changes are natural candidates for wind shear applications.

State Space Model

In view of the above discussion, let us assume that the radar return can be modeled as

$$r_t = h(s_t) + v_t \tag{1}$$

where $h(s_t) = A \sin(\omega_c + 4\pi s_t/\lambda)t$,
- A = amplitude (envelope),
- s_t = radial velocity (along the radar's beam axis),
- v_t = Gaussian white noise with zero mean and correlation matrix V_c,
- ω_c = carrier frequency, and
- λ = wavelength.

We are thus assuming a phase modulation format where each measurement r_t represents a return at some fixed range gate; other formats might also be applicable. More precisely, consider the following figure denoting radar returns from sequential pulses of pulse repetition time T_s and round trip propagation time τ_s as defined above:

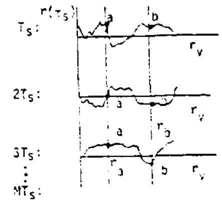

Figure 1

The sequence of M points {a} denotes samples of the scattered signal at some distance r_a in the entire volume, {b} denotes samples at distance r_b, etc. Imagine, for example, that each return is separated by $T_s = 0.1$ ms arising from an assumed microburst of 15 km in depth. These various sequences can be used to generate statistics for s_t at the corresponding ranges r_v within the microburst. For example, one could use all M samples in a batch mode to compute the radial velocity with a resolution $(\lambda/2)(1/MT_s)$, where λ is the radar carrier wavelength; either FFT or covariance methods could be used.

Now suppose one wishes to estimate the occurrence of the abrupt change in the velocity s_t that accompanies a microburst. Consider the following figure:

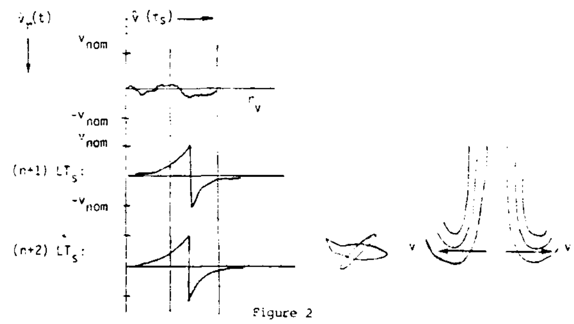

Figure 2

Here we have shown that, in a microburst, the downburst impacts the ground and the magnitude of the velocity v changes abruptly from zero at the center of the microburst to some relatively large nominal value $\pm v_{nom}$; in the absence of a microburst, the magnitude and direction of v are random.

To formulate a state space model for the above situation, let us imagine that before the microburst, the velocity is a stochastic process x_t whose mean is a relatively small nonzero constant and whose variance is a relatively large constant; after the microburst, we shall imagine that the velocity is a stochastic process b_t whose mean is a relatively large constant (corresponding to v_{nom}) and whose variance is relatively small. Thus we can write

$$s_t = b_t z_t + x_t (1-z_t) \tag{2}$$

where

$$z_t = \begin{cases} 0 & \text{if } t < T \\ 1 & \text{if } t \geq T \end{cases} \tag{3}$$

and T denotes the random time of occurrence of the microburst. If we now assume that T is an exponentially distributed random variable with parameter λ, and redefine the observations r_t in (1) as $dy_t = r_t dt$, then a possible model might be

$$\text{state:} \quad \begin{bmatrix} dx \\ dz \\ db \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda \\ 0 \end{bmatrix} dt + \begin{bmatrix} -F & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -G \end{bmatrix} \begin{bmatrix} x \\ z \\ b \end{bmatrix} dt + \begin{bmatrix} dW_t \\ dM_t \\ dB_t \end{bmatrix} \qquad \begin{matrix} (4\text{-a}) \\ (4\text{-b}) \\ (4\text{-c}) \end{matrix}$$

$$\text{observation:} \quad dv_t = A \sin(\omega_c + 4\pi[b_t z_t + x_t(1-z_t)]/\lambda)t + dV_t \tag{5}$$

where W_t = Brownian motion (mean zero and variance $W_c t$),
M_t = martingale associated with a Poisson process z_t stopped at its first jump time,
B_t = Brownian motion (mean zero and variance $B_c t$),
V_t = Brownian motion (mean zero and variance $V_c t$), and
all Brownian motions and initial conditions are mutually independent.

The parameters F and G must be chosen to achieve the desired statistical properties mentioned above and, at the same time, must guarantee stable solutions when their discrete approximations are implemented on a digital computer. The stability issue will be discussed later in the section on simulation. Considering the statistical properties required of x_t, its solution, with x_o assumed deterministic, implies the following mean and variance:

$$Ex_t = e^{-Ft}x_o, \quad \text{var } x_t = (W_c/2F)(1-e^{-2Ft}).$$

Since in reality we are unable to have $t \to \infty$ to obtain a large constant variance, we shall choose F to be relatively large (and W_c even larger). However, such a choice drives the mean to zero, which is not desired. Therefore, to compensate for this, the x_t used in (5) will actually be the x_t from (4-a) plus a constant nominal value of x_{nom}. On the other hand, since the b_t process has a mean and variance with the same form as that for x_t, and the desired statistical properties of b_t are opposite those of x_t, we choose G to be relatively small.

Filtering and Detection Algorithm

Using the state and observation equations in (4) and (5) above, we can generate MMSE estimates $\hat{x}$, $\hat{z}$, and $\hat{b}$ of the respective states x, z, and b and use them to detect the onset of wind shear. More specifically, recalling the definition of z_t in (3), it turns out that

$$\hat{z}_t = \Pr[t \geq T | y_s, 0 \leq s \leq t] \tag{6}$$

i.e., the conditional mean $\hat{z}_t$ is precisely the probability that wind shear has occurred. However, because our model represents a system of nonlinear stochastic differential equations, it is not possible to derive the optimal estimate $\hat{z}$ in terms of a finite-dimensional realization. Therefore, this so-called moment closure problem compels us to seek a suboptimal filter. Once specified, then we can use the suboptimal estimate $\bar{z}_t$ of z_t to generate an estimate $\bar{T}$ of T, the first time of occurrence of a microburst, by choosing some threshold $k \in (0,1)$ and setting

$$\bar{T} = \min\{t | \bar{z}_t \geq k\}. \tag{7}$$

It is the form of (7) that causes us to describe the problem as one of first-passage. Its successful use as a detection algorithm will depend heavily on the quality of the estimates of z provided by the filter; the k parameter is also important in that it is related to the performance issue and the concomitant false-alarm and miss error-probabilities. With this in mind, we have opted to compare the use of the extended Kalman filter (EKF) with the truncated second order filter (TSOF) as presented, for example, by Jazwinski [6,chap 9]. A slight modification in the error-covariance equations, however, is required because of the martingale M_t in (4-b) associated with the purely discontinuous process z_t. In any case, with overbars denoting suboptimal estimates and P denoting the error-covariance matrix, we have the following:

$$\begin{bmatrix} d\bar{x} \\ d\bar{z} \\ d\bar{b} \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda \\ 0 \end{bmatrix} dt + \begin{bmatrix} -F & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -G \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{z} \\ \bar{b} \end{bmatrix} dt + P \begin{bmatrix} \partial\bar{h}/\partial\bar{x} \\ \partial\bar{h}/\partial\bar{z} \\ \partial\bar{h}/\partial\bar{b} \end{bmatrix} V_c^{-1} \cdot IN \tag{8}$$

$$dP = \left\{ \begin{bmatrix} -F & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -G \end{bmatrix} P + P \begin{bmatrix} -F & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -G \end{bmatrix} + \begin{bmatrix} W_c & 0 & 0 \\ 0 & \lambda(1-\bar{z}) & 0 \\ 0 & 0 & B_c \end{bmatrix} - V_c^{-1} P \begin{bmatrix} \partial\bar{h}/\partial\bar{x} \\ \partial\bar{h}/\partial\bar{z} \\ \partial\bar{h}/\partial\bar{b} \end{bmatrix} \begin{bmatrix} \partial\bar{h}/\partial\bar{x} \\ \partial\bar{h}/\partial\bar{z} \\ \partial\bar{h}/\partial\bar{b} \end{bmatrix}^T P \right\} dt - \frac{1}{2} V_c^{-1} P \cdot SOT \cdot IN \tag{9}$$

where $IN \triangleq dy - (\bar{h} + \frac{1}{2} \cdot SOT)dt$, innovations
$SOT \triangleq \text{trace } [PH]$, second-order-term, and
$H \triangleq \partial^2\bar{h}/\partial(\bar{\cdot})^2$, Hessian matrix.

In deriving these equations, use is made of the fact that the definition of z_t in (3) implies $z_t^2 = z_t$. Also the term $\lambda(1-\bar{z})$ in the error-covariance equation is a result of the quadratic variation of the martingale M_t in (4-b). Finally note that if the term SOT is zero, then the preceding equations define the EKF; otherwise, they define the TSOF.

3. SIMULATION

The approach used in discretizing the continuous-time equations given in the previous section is based on Euler's approximation; see, for example, Franklin and Powell [7, ch 3]. Since the equations in (4) are decoupled, we can illustrate the approach by discretizing (4-a) to obtain

$$x_{k+1} = (1 - F\Delta)x_k + w_k\Delta$$

where w_k is a zero-mean Gaussian white noise sequence with correlation matrix W_c. To be stable, one requires that $|1-F\Delta|<1$, i.e., $0 < F < 2/\Delta$ for a sampling interval of length Δ which, for us, has been chosen to be 1. Analogous statements hold for (4-c). Therefore, recalling the discussion on parameter selection in the previous section on state-space models, we choose $F \approx 1.90$ and $G \approx .01$.

The results obtained thus far from computer simulations have been encouraging and lead us to conclude that the overall approach is a sound one. As is so often the case in such simulations, care must be taken in selecting values of state and observation noise statistics and initial error-covariances to prevent filter divergence, and we are continuing to investigate adaptive techniques to remedy this problem. However, in those cases where the filtered estimates did converge, the decision that wind shear had occurred was made within a few sampling intervals of the actual occurrence. It is anticipated that a complete presentation of these simulations will be forthcoming in the near future.

We also plan to simulate microbursts through the use of a model based on the fluid continuity equation

$$\nabla \cdot (\rho v) = 0$$

with vertical hydrostatic equilibrium. Assumptions include inviscid flow with no heat input (dry microburst). An azimuthally symmetric solution in cylindrical coordinates, with v_z and v_r denoting vertical and radial velocity components, respectively, is

$$v_z = \frac{bc}{1+bH} e^{-r^2/2A^2}(e^{-z/b} - e^{-Hz})$$

$$v_r = \frac{2A^2c}{2} \frac{1 - e^{-r^2/2A^2}}{r} e^{-z/b}$$

Here, H is the actual scale height, A is the initial radius at the top of the column, c is the initial velocity or strength, and b is a parameter affecting the shape (outflow height). Through a slight modification, the model can accommodate asymmetrical microbursts with an essentially arbitrary vertical shear profile, the addition of a vortex ring to more closely simulate an actual microburst, and wet microbursts. Given the present state of knowledge concerning wind shear, this model produces geometries that are in excellent agreement with those that have been actually observed. Futhermore, with this more realistic model, we shall be able to simulate radar returns from several different range gates.

4. IMPLEMENTATION

In implementing the actual system, several essential parameters regarding the hardware and data acquisition scheme will have to be determined. These include items such as range and antenna weighting functions, antenna gain, size of the resolution volume scanned by the beam, receiver bandwidth, and pulse repetition time. In this paper we shall only discuss the latter where, ideally, it should be chosen long enough so that returns from consecutive pulses do not overlap and yet short enough so that samples obtained at one range are well correlated for information extraction purposes. We now elaborate.

Since the system studied here will be used in airborne applications, the processing rate, and thus the alert time, is a crucial factor. The processing rate has to be fast enough so that the pilot can be warned well in advance to avoid wind shear. This rate can be improved by overlapping computation time with scanning/sampling time. In this context, the system is thought of as two separate units. The first is a radar that is continuously transmitting and receiving signals and a sampler that is sampling the received signals and storing them in memory banks according to their range as shown in Fig. 3. The second unit is a processor that is computing estimates and making decisions. While the processor is processing returns from the first M pulses, the returns of the second M pulses are being sampled and stored in memory. In order to do this overlapping, the processor must process the data at a rate higher or at least equal to the rate at which

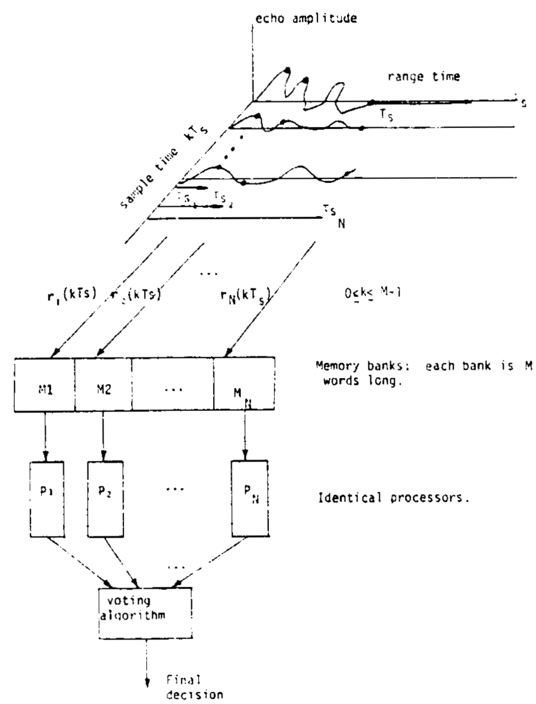

Figure 3

the data are received. For example, if $M = 256$ and $T_s = .1ms$, then this rate is 1/25.6 kHz. Such a high rate may not (depending on the value of N) be achieved with a serial processor. Instead, one can use a number of identical processors that run in a parallel fashion; see Figure 3. Each processor is small, fast, and responsible for all the computation required at one range gate. The computation consists of spectral estimation, filtering, and abrupt jump monitoring. In addition to achieving a high processing rate, this scheme is advantageous in that N decisions, regarding the presence or absence of wind shear, at N range gates will be made all at once. This is a very informative aspect because in a way it reflects the dimension of a microburst along the beam axis and it allows for rejection of false alarms on part of the individual processors, i.e., a microburst should trigger the output of more than one processor. Thus, a voting algorithm, based on prespecified criteria, can be implemented to produce one final decision.

A large number of range time samples, however, imposes a limitation on our processing scheme. A large N requires a large number of processors which could be impractical due to space and power limitations on board an aircraft. In such a case the N discrete sequences can be divided into groups each of which is J sequences long, where J is a submultiple of N. Every group will be processed in a parallel fashion on J processors, while the different groups are fed serially. Even in this mixed processing, an important improvement in the processing rate would be achieved.

5. CONCLUSION

The preliminary findings described above strongly suggest that the first-passage analysis we have adopted provides a viable approach to the problem of identifying the onset of wind shear. The results to date center on an idealized symmetric geometry for the wind pattern and have neglected noise processes specifically related to ground clutter. It is not expected, however, that the relaxation of these constraints will alter the basic conclusion. Future work will incorporate real data and thus will be able to determine the ultimate efficacy of the approach.

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